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Supersymmetric Contributions to Weak Decay Correlation Coefficients

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Abstract

We study supersymmetric contributions to correlation coefficients that characterize the spectral shape and angular distribution for polarized μ- and β-decays. In the minimal supersymmetric Standard Model (MSSM), one-loop box graphs containing superpartners can give rise to non-$(V-A)\otimes(V-A)$ four fermion operators in the presence of left-right or flavor mixing between sfermions. We analyze the present phenomenological constraints on such mixing and determine the range of allowed contributions to the weak decay correlation coefficients. We discuss the prospective implications for future μ- and β-decay experiments, and argue that they may provide unique probes of left-right mixing in the first generation scalar fermion sector.

1 Introduction

The search for physics beyond the Standard Model (SM) lies at the forefront of particle and nuclear physics. Among the prospective candidates for SM extensions, low-energy supersymmetry (SUSY) remains one of the most attractive possibilities. Its elegant solution to the naturalness problem associated with stability of the electroweak scale, its generation of coupling unification near the GUT scale, and its viable particle physics explanations for the abundance of matter (both visible and dark), have motivated a plethora of phenomenological studies over the years. With the advent of the Large Hadron Collider (LHC), direct evidence for low-energy SUSY may become available in the near future.

In the search for new physics, studies of precision electroweak observables and rare or SM-forbidden processes provide an important and complementary probe to collider searches (for recent discussions, see Refs. [1, 2]). Indeed, precise measurements of Z-pole observables and other electroweak precision measurements, as well as the branching ratios of rare decays such as $b \to s\gamma$ or $B_s \to \mu^+\mu^-$ have placed important constraints on supersymmetric models. At low energies, the recent evidence for a possibly significant deviation of the muon anomalous magnetic moment, $(g_\mu - 2)$, from SM expectations provides at least a tantalizing hint of low-energy SUSY in the regime of large $\tan\beta$ [3,4]. Similarly, new searches for the permanent electric dipole moments of various systems will probe SUSY (and other) CP-violation sources at a level of interest to explaining the baryon asymmetry of the universe [1, 5], while precision studies of fixed target, parity-violating electron scattering will be strongly sensitive to the existence lepton-number violating supersymmetric interactions [6].

In this paper, we study the implications of SUSY for weak decays of the muon, neutron, and nuclei. Our work is motivated by the prospect of significantly higher precision in future measurements of the muon lifetime (τ_μ) and decay correlation parameters, as well as of considerably higher precision in studies of neutron and nuclear β-decay at various laboratories, including the Spallation Neutron Source (SNS), Los Alamos Neutron Science Center (LANSCE), NIST, the Institut Laue-Langevin (ILL), and a possible high-intensity radioactive ion beam facility. Recent experimental progress including: new measurements of muon decay parameters at TRIUMF and PSI [7–9]; new measurements of the muon lifetime at PSI [10, 11]; Penning trap studies of nuclear β-decay [12]; the development of cold and ultracold neutron technology for the study of neutron decay; and plans for new measurements of the leptonic decays of the pion [13, 14] – point to the high level of experimental activity in this direction.

Theoretically, recent efforts have focused on the use of such experiments to test the unitarity of the Cabibbo-Kobayashi-Maskawa matrix, including analyses of SUSY corrections to the $(V-A)\otimes(V-A)$ structure of the SM charged current (CC) interaction [15–17] and improved limits on hadronic structure effects in neutron and nuclear β-decays [18]. Going beyond the $(V-A)\otimes(V-A)$ structure of the SM CC interaction, it has recently been shown that the scale of neutrino mass implied by neutrino oscillation experiments and the cosmic microwave background implies stringent bounds on chiralitychanging scalar and tensor operators that could contribute to weak decays [19–21]. Comprehensive reviews of non-$(V-A)\otimes(V-A)$ effects in β-decay have been given in Refs. [22–24]

Here, we study the effects of supersymmetric interactions that give rise to non-$(V-A)\otimes(V-A)$ interactions but evade the neutrino mass bounds. Such effects can arise through radiative corrections in supersymmetric models containing only left-handed neutrinos. For concreteness, we focus on the minimal supersymmetric Standard Model (MSSM). We do not consider simple extensions of the MSSM with right-handed or Majorana neutrinos and their superpartners as required by non-vanishing neutrino mass, as the effects of the corresponding neutrino sector on weak decays is highly constrained.[1]. We show that in the MSSM, radiatively-induced non-$(V-A)\otimes(V-A)$ interactions are particularly sensitive to flavor and left-right mixing among first and second generation sleptons and squarks. The flavor structure of the MSSM has been the subject of extensive scrutiny, and as we show below, experimental studies of lepton flavor violation (LFV) lead to tight constraints on the corresponding effects in weak decays. In contrast, there exist few independent probes of left(L)-right(R) mixing among scalar fermions. It is generally assumed that this mixing is proportional to the relevant Yukawa couplings, implying that it is highly suppressed in processes involving only first and second generation sfermions. However, this "alignment" assumption is not inherent in the structure of the SUSY–breaking sector of the MSSM, and while its use can simplify MSSM phenomenology, it is of interest to explore experimental observables that may test this assumption. In what follows, we argue that studies of weak decay correlations may provide such experimental tests. Specifically, we find that:

(i) Supersymmetric box graph corrections to the μ-decay amplitude generates a non-vanishing scalar interaction involving right handed charged leptons ($g^S_{RR}\neq 0$ in the standard parameterization used below) in the presence of flavor mixing among left-handed (LH) sneutrinos and among right-handed (RH) sleptons, or flavor-diagonal mixing among LH and RH sleptons.

(ii) Analogous box graph effects can give rise to non-vanishing scalar and tensor interactions in light quark β-decay. The generation of these interactions requires non-vanishing left-right mixing among first generation sleptons and squarks. Studies of the energy-dependence of β-decay correlations and β-polarization – as well as of the energy-independent spin-polarization correlation – provide a probe of these interactions and the requisite L-R mixing.

(iii) Flavor-mixing among the LH sleptons ($\tilde{\nu}_L$, $\tilde{\ell}_L$) is highly constrained by searches for lepton flavor violation in processes such as $\mu\to e\gamma$ and $\mu\to e$ conversion. Thus, any observable departure from $(V-A)\otimes(V-A)$ interactions in μ-decay associated with the MSSM would arise from large, flavor diagonal L-R mixing among smuons ($\tilde{\mu}$) and selectrons ($\tilde{e}$). The former are also

[1] In see-saw scenarios, for example, the scale of the additional sneutrino mass is sufficiently large that these degrees of freedom decouple from low-energy observables.

constrained by the present value of $(g_\mu - 2)$. At present, we are not aware of any analogous constraints on L-R mixing among first generation squarks and sleptons.

(iv) The magnitude of the effects in μ-decay are below the present sensitivity of decay correlation studies. However, the presence of the SUSY-induced scalar interaction could modify the extraction of the Fermi constant (G_μ) from the next generation of τ_μ measurements at PSI. Similarly, improvements in β-decay correlation precision by $\lesssim$ an order of magnitude would allow one to probe the SUSY-induced scalar and tensor interactions generated by large L-R mixing. Such measurements could in principle provide a unique test of L-R mixing among first generation superpartners.

In the remainder of the paper, we provide details of our analysis. Section 2 gives a general overview of weak decay correlations and our notation and conventions. In Section 3 we discuss the computation of the relevant SUSY corrections and give analytic expressions for the resulting operators. Section 4 contains a discussion of constraints resulting from other measurements and numerical implications for the μ-decay and β-decay correlations. We summarize our conclusions in Section 5.

2 Weak Decay Correlations: General Features

Departures from the SM $(V-A)\otimes(V-A)$ structure of the low-energy leptonic and semileptonic CC weak interactions can be characterized by an effective four fermion Lagrangian containing all independent dimension six operators. In the case of μ-decay, it is conventional to use

$$\mathcal{L}^{\mu-\mathrm{decay}} = -\frac{4G_\mu}{\sqrt{2}} \sum_{\gamma,\,\epsilon,\,\mu} g^{\gamma}_{\epsilon\mu}\ \bar{e}_\epsilon \Gamma^\gamma \nu_e\, \bar{\nu}_\mu \Gamma_\gamma \mu_\mu \tag{1}$$

where the sum runs over Dirac matrices $\Gamma^\gamma = 1$ (S), γ^α (V), and $\sigma^{\alpha\beta}/\sqrt{2}$ (T) and the subscripts μ and ϵ denote the chirality (R,L) of the muon and final state lepton, respectively[2]. Note that the use of this Lagrangian is only appropriate for the analysis of processes that occur at energies below the electroweak scale, as it is not $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ invariant. At tree-level in the SM one has $g^V_{LL} = 1$ with all other $g^\gamma_{\epsilon\mu} = 0$. In the limit of vanishing lepton masses, non-QED SM electroweak radiative corrections to the tree-level amplitude are absorbed into the definition of G_μ.

In the literature, there exist several equivalent parameterizations of non-Standard Model contributions to light quark β-decay [22–24]. In analogy with Eq. (1) we use

$$\mathcal{L}^{\beta-\mathrm{decay}} = -\frac{4G_\mu}{\sqrt{2}} \sum_{\gamma,\,\epsilon,\,\delta} a^{\gamma}_{\epsilon\delta}\ \bar{e}_\epsilon \Gamma^\gamma \nu_e\, \bar{u} \Gamma_\gamma d_\delta \tag{2}$$

[2]The normalization of the tensor terms corresponds to the convention adopted in Ref. [25]

where the notation is similar to that for the μ-decay effective Lagrangian. As with $\mathcal{L}^{\mu-\mathrm{decay}}$, only the purely left-handed $(V-A)\otimes(V-A)$ interaction appears at tree-level in the SM. In this case, one has $a^V_{LL} = V_{ud}$, the (1,1) element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Including electroweak radiative corrections leads to

$$a^V_{LL} = V_{ud}\,\left(1 + \Delta\widehat{r}_\beta - \Delta\widehat{r}_\mu\right) \quad , \tag{3}$$

where $\Delta\widehat{r}_\beta$ contains the electroweak radiative corrections to the tree-level $(V-A)\otimes(V-A)$ β-decay amplitude and $\Delta\widehat{r}_\mu$ contains the corresponding corrections for μ-decay apart from the QED corrections that are explicitly factored out when definiting G_μ from the muon lifetime.

Supersymmetric contributions to $\Delta\widehat{r}_\beta - \Delta\widehat{r}_\mu$ have been computed in Refs. [15–17]. These corrections can affect tests of the unitarity of the first row of the CKM matrix, as they must be subtracted from a^V_{LL} when determining V_{ud} from β-decay half lives. The corresponding implications for CKM unitarity tests have been discussed in those studies. Supersymmetric corrections can also give rise to non-vanishing $g^\gamma_{\epsilon\mu}$ and $a^\gamma_{\epsilon\delta}$ that parameterize the non-$(V-A)\otimes(V-A)$ interactions in Eqs. (1,2). The presence of these operators cannot be discerned using the muon lifetime or β-decay half lives alone, but they can be probed using studies of the spectrum, angular distribution, and polarization of the decay products. We consider here non-$(V-A)\otimes(V-A)$ that are generated by one-loop corrections and are, thus, suppressed by a factor of $\alpha/4\pi$. Consequently, we focus on those operators that interfere linearly with SM contributions in weak decay observables and have the largest possible phenomenological effects.

In the case of polarized μ^- (μ^+) decay, the electron (positron) spectrum and polarization are characterized by the eleven Michel parameters [26,27], four of which (ρ, η, ξ, and δ) describe the spectral shape and angular distribution. An additional five (ξ', ξ'', η'', α/A, β/A) are used to characterize the electron (positron) transverse and longitudinal polarization, while the final two (α'/A, β'/A) parameterize time-reversal odd correlations between the muon polarization and the outgoing charged lepton spin. In what follows, we find that SUSY box graphs generate non-vanishing contributions to g^S_{RR}. This coupling appears quadratically in the parameters ξ and ξ' and interferes linearly with the SM term g^V_{LL} in η, η'', and β'/A. The linear-dependence of η on g^S_{RR} is particularly interesting, since $\eta = 0$ in the SM, and since a non-zero value for this parameter enters the extraction of G_μ from τ_μ:

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[1+\delta_{\mathrm{QED}}\right]\left[1 + 4\eta\frac{m_e}{m_\mu} - 8\left(\frac{m_e}{m_\mu}\right)^2\right]\left[1+\frac{3}{5}\left(\frac{m_\mu}{M_W}\right)^2\right] \quad , \tag{4}$$

where δ_{QED} denote the QED corrections the decay rate in the low-energy (Fermi) effective theory.

For β-decay, it is customary to use the description of the differential decay rate written down by

Jackson, Treiman, and Wyld [28] (see also [22–24]):

$$
\begin{aligned}
d\Gamma \quad &\propto \quad \mathcal{N}(E_e)\Bigg\{1 + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_eE_\nu} + b\frac{\Gamma m_e}{E_e} + \langle\vec{J}\rangle\cdot\left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e\times\vec{p}_\nu}{E_eE_\nu}\right] \\
&+\vec{\sigma}\cdot\left[N\langle\vec{J}\rangle + G\frac{\vec{p}_e}{E_e} + Q'\hat{p}_e\hat{p}_e\cdot\langle\vec{J}\rangle + R\langle\vec{J}\rangle\times\frac{\vec{p}_e}{E_e}\right]\Bigg\}d\Omega_e d\Omega_\nu dE_e,
\end{aligned}
\tag{5}
$$

where $\mathcal{N}(E_e) = p_eE_e(E_0-E_e)^2$; E_e (E_ν), $\vec{p}_e$ $(\vec{p}_\nu)$, and $\vec{\sigma}$ are the β (neutrino) energy, momentum, and polarization, respectively; $\vec{J}$ is the polarization of the decaying nucleus; and $\Gamma = \sqrt{1-(Z\alpha)^2}$.

As we discuss below, SUSY box graphs may generate non-zero contributions to the operators in Eq. (2) parameterized by a^S_{RR}, a^S_{RL}, and a^T_{RL}. The latter interfere linearly with the SM parameter a^V_{LL} in terms in Eq. (5) that depend on β energy, including the so-called Fierz interference coefficient, b; the parity-violating correlation involving neutrino momentum and nuclear spin, B; and the polarization correlation coefficient, Q'. In addition, the energy-independent spin-polarization correlation coefficient N also contains a linear dependence on a^S_{RR}, a^S_{RL}, and a^T_{RL}. Specifically, one has

$$
b\,\zeta = \pm\ 4\,\mathrm{Re}\left[M_F^2\, g_Vg_S\, a^V_{LL}(a^S_{RL}+a^S_{RR})^* - 2M_{GT}^2\, g_Ag_T\, a^V_{LL}a^{T*}_{RL}\right] \tag{6}
$$

$$
\begin{aligned}
B\,\zeta = 2\,\mathrm{Re}\Bigg[&\pm\lambda_{J'J}\, M_{GT}^2\left((g_A^2\,|a^V_{LL}|^2 + 4\,g_T^2\,|a^T_{RL}|^2)\ \mp\ \frac{\Gamma m}{E}\,4\,g_Ag_T\,a^V_{LL}a^{T*}_{RL}\right) \\
&+\delta_{J'J}\,M_FM_{GT}\sqrt{\frac{J}{J+1}}\,\Big(2\,g_Vg_A\,|a^V_{LL}|^2 \\
&\qquad\mp\ \frac{\Gamma m}{E}\,(4\,g_Vg_T\,a^V_{LL}a^{T*}_{RL} - 2\,g_Ag_S\,a^V_{LL}(a^S_{RL}+a^S_{RR})^*)\ \Big)\Bigg]
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
Q'\,\zeta = 2\,\mathrm{Re}\Bigg[&\lambda_{J'J}\, M_{GT}^2\left((g_A^2\,|a^V_{LL}|^2 + 4g_T^2\,|a^T_{RL}|^2)\ \mp\ \frac{\Gamma m}{E}\,2\,g_Ag_T\,a^V_{LL}a^{T*}_{RL}\right) \\
&\mp\ \delta_{J'J}\,M_FM_{GT}\sqrt{\frac{J}{J+1}}\,\Big(2\,g_Vg_A\,|a^V_{LL}|^2 \\
&\qquad\mp\ \frac{\Gamma m}{E}\,(4\,g_Vg_T\,a^V_{LL}a^{*T}_{RL} - 2\,g_Ag_S\,a^V_{LL}(a^S_{RL}+a^S_{RR})^*)\ \Big)\Bigg]
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
N\,\zeta = 2\,\mathrm{Re}\Bigg[&\lambda_{J'J}\, M_{GT}^2\left(\frac{\Gamma m}{E}\,(4g_T^2\,|a^T_{RL}|^2 + g_A^2\,|a^V_{LL}|^2)\mp 4g_Tg_A\,a^T_{RL}a^{V*}_{LL}\right) \\
&+\ \delta_{J'J}\,M_FM_{GT}\sqrt{\frac{J}{J+1}}\,\Big((4g_Vg_T\,a^V_{LL}a^{T*}_{RL} - 2g_Sg_A\,(a^S_{RR}+a^S_{RL})a^{V*}_{LL}) \\
&\qquad\pm\ \frac{\Gamma m}{E}\,(4g_Sg_T\,(a^S_{RR}+a^S_{RL})a^{T*}_{RL} - 2g_Vg_A\,|a^V_{LL}|^2)\ \Big)\Bigg]
\end{aligned}
\tag{9}
$$

$$
\zeta = 2M_F^2\,\left(g_V^2\,|a^V_{LL}|^2 + g_S^2\,|a^S_{RL}+a^S_{RR}|^2\right) + 2M_{GT}^2\,(g_A^2\,|a^V_{LL}|^2 + 4\,g_T^2\,|a^T_{RL}|^2)\quad, \tag{10}
$$

$$
\tag{11}
$$

where J (J') are the initial (final) nuclear spin and $\lambda_{J'J} = (1, 1/J+1, -J/J+1)$ for $J' = (J-1, J, J+1)$[3]. The quantities M_F and M_{GT} are Fermi and Gamow-Teller matrix elements and $g_{V,A,S,T}$

[3]The quantity ζ is often denoted by ξ in the literature. However, we have modified the notation to avoid confusion

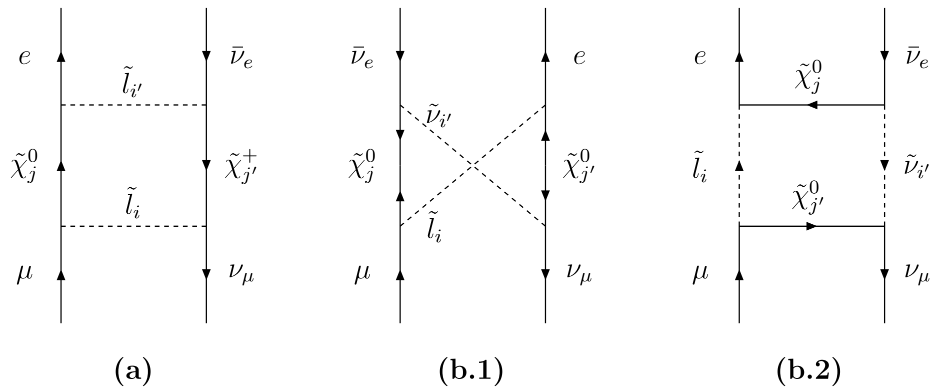


Figure 1: *Feynman diagrams relative to supersymmetric contributions giving rise to non $(V-A)\otimes(V-A)$ amplitudes in the muon decay. The amplitude relative to the diagram shown in* **(a)** *involves left-right slepton mixing, while those in* **(a)** *are non-vanishing if lepton flavor mixing is present in the slepton sector.*

are vector, axial vector, scalar, and tensor form factors. For transitions between initial (i) and final (f) nuclear states the corresponding reduced matrix elements are

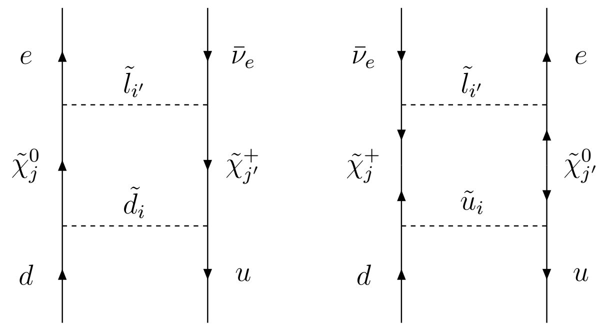
$$\begin{aligned}
\langle f||\bar{u}\gamma^\lambda d + \mathrm{H.C.}||i\rangle &= g_V(q^2)M_F \\
\langle f||\bar{u}\gamma^\lambda \gamma_5 d + \mathrm{H.C.}||i\rangle &= g_A(q^2)M_{GT} \\
\langle f||\bar{u}d + \mathrm{H.C.}||i\rangle &= g_S(q^2)M_F \\
\langle f||\bar{u}\sigma^{\lambda\rho} d + \mathrm{H.C.}||i\rangle &= g_T(q^2)M_{GT} \quad .
\end{aligned} \tag{12}$$

The conserved vector current property of the SM CC interaction implies that $g_V(0) = 1$ in the limit of exact isospin symmetry. Isospin breaking corrections imply deviations from unity of order a few $\times 10^{-4}$ [29] (for an earlier estimate, see Ref. [30]). A two parameter fit to β-decay data yields $g_A(0)/g_V(0) = 1.27293(46)$ [24], assuming only a non-vanishing SM coupling, a^V_{LL}, and neglecting differences in electroweak radiative corrections between hadronic vector and axial vector amplitudes. Theoretical expectations for g_S and g_T are summarized below.

3 SUSY-Induced Scalar and Tensor Interactions

We compute the SUSY contributions to the weak decay correlations in the MSSM and obtain non-vanishing contributions to g^S_{RR}, a^S_{RR}, and $a^{S,T}_{RL}$ from the box diagrams in Figures 1 (μ-decay) and 2 (β-decay). These amplitudes – as well as others not shown explicitly in Figures 1 and 2 – also contribute to the parameters g^V_{LL} and a^V_{LL} that arise in the SM. A complete analysis of those contributions, along with the gauge boson propagator, vertex, and external leg corrections, is given in Ref. [15]. Here, we focus on the non-$(V-A)\otimes(V-A)$ operators generated by the diagrams shown explicitly.

with the Michel parameter ξ.



(a) **(b)**

Figure 2: *Feynman diagrams relative to supersymmetric contributions giving rise to anomalous amplitudes in β decay processes.*

Since the SM CC interaction is purely left-handed (LH), the generation of operators involving the right-handed (RH) SM fields requires the presence of RH fermion superpartners in the one-loop graphs. These particles appear either by virtue of left-right mixing among superpartners in Figures 1(a) and 2(a,b) or through coupling of the neutralinos ($\tilde{\chi}_j^0$) to the RH sleptons as in Figures 1(b.1, b.2). Note that in the latter case, a given virtual slepton mass eigenstate $\tilde{l}_i$ will couple to both the first and second generation charged leptons, thereby requiring the presence of non-zero flavor mixing. In contrast, the contributions of Figures. 1(a) and 2(a,b) involve only L-R mixing but no flavor mixing among the sfermions in the loops.

To set our notation, we largely follow the conventions of Refs. [31,32]. The L-R and flavor mixing among the sfermions is determined by the sfermion mass matrices. In the flavor basis, one has

$$\mathbf{M}^2_{\tilde{\mathbf{f}}} = \begin{pmatrix} \mathbf{M}^2_{\mathbf{LL}} & \mathbf{M}^2_{\mathbf{LR}} \\ \mathbf{M}^2_{\mathbf{LR}} & \mathbf{M}^2_{\mathbf{RR}} \end{pmatrix} \tag{13}$$

where for quark and charged slepton superpartners $\mathbf{M}^2_{\mathbf{AB}}$ $(A, B = L, R)$ are 3×3 matrices with indices running over the three flavors of sfermion of a given chiral multiplet (L, R). For sneutrinos, only $\mathbf{M}^2_{\mathbf{LL}}$ is non-vanishing. After electroweak symmetry-breaking, the $\mathbf{M}^2_{\mathbf{AB}}$ take the forms (using squarks as an illustration)

$$\mathbf{M}^2_{\mathbf{LL}} = \mathbf{m}^2_{\mathbf{Q}} + \mathbf{m}^2_{\mathbf{q}} + \mathbf{\Delta_f} \tag{14}$$

$$\mathbf{M}^2_{\mathbf{RR}} = \mathbf{m}^2_{\tilde{\mathbf{f}}} + \mathbf{m}^2_{\mathbf{q}} + \bar{\mathbf{\Delta}}_{\mathbf{f}} \tag{15}$$

with

$$\mathbf{\Delta_f} = \left(I_3^f - Q_f \sin^2\theta_W\right)\ \cos 2\beta M_Z^2 \tag{16}$$

$$\bar{\mathbf{\Delta}}_{\mathbf{f}} = Q_f \sin^2\theta_W\ \cos 2\beta M_Z^2 \tag{17}$$

and

$$\mathbf{M^2_{LR}} = \mathbf{M^2_{RL}} = \begin{cases} v\left[\mathbf{a_f}\sin\beta - \mu\mathbf{Y_f}\cos\beta\right] \ , & \tilde{u} - \text{type sfermion} \\ v\left[\mathbf{a_f}\cos\beta - \mu\mathbf{Y_f}\sin\beta\right] \ , & \tilde{d} - \text{type sfermion} \end{cases} \quad , \tag{18}$$

where $\tan\beta = v_u/v_d$ gives the ratio of the vacuum expectation value of the two neutral Higgs fields, $\mathbf{Y_f}$ and $\mathbf{a_f}$ are the 3×3 Yukawa and soft triscalar couplings and μ is the supersymmetric coupling between the two Higgs supermultiplets. The matrices $\mathbf{m^2_Q}$, $\mathbf{m^2_{\tilde{f}}}$, and $\mathbf{m^2_q}$ are the mass matrices for the LH squarks, RH squarks, and quarks, respectively. It is often customary to assume that $\mathbf{a_f} \propto \mathbf{Y_f}$, in which case one may diagonalize $\mathbf{M^2_{LR}}$ by the same rotation that diagonalizes the fermion mass matrices and leads to a magnitude for L-R mixing proportional to the relevant fermion mass. In what follows, however, we will avoid making this assumption. Indeed, studies of the decay correlation parameters may provide a means of testing this alignment hypothesis.

The matrix $\mathbf{M^2_{\tilde{f}}}$ can be diagonalized by the unitary matrix $\mathbf{Z_f}$. The corresponding sfermion mass eigenstates $\tilde{F}_j$ are given as a linear combination of the flavor eigenstates $\tilde{f}_I$ as

$$\tilde{F}_j = Z_f^{jI}\,\tilde{f}_I \tag{19}$$

where $I = 1,2,3$ indicate the flavor states $\tilde{f}_{L_I}$ and $I = 4,5,6$ refer to the RH flavor states $\tilde{f}_{R_{I-3}}$.

In general, the charginos $(\tilde{\chi}_j^+)$ and neutralinos entering the loop graphs are mixtures of the electroweak gauginos and Higgsinos. Since the characteristics of this mixing are not crucial to our analysis and a detailed discussion can be found elsewhere (*e.g.*, Refs. [31,32]), we simply give our notation for the relevant mixing:

$$\chi_i^0 = N_{ij}\psi_i^0 \quad i,j = 1\ldots 4 \tag{20}$$

for the neutralinos and

$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix} = \mathbf{V}\begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} \quad , \quad \begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix} = \mathbf{U}\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}; \tag{21}$$

for the charginos. Here, the fields ψ_i^0 denote the $(\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$ fields.

Using the foregoing conventions, we obtain the following contributions to the g^S_{RR}, a^S_{RR}, and $a^{S,T}_{RL}$:

$$\begin{aligned} g^S_{RR} &= \tilde{\delta}_\mu^{(a)} + \tilde{\delta}_\mu^{(b.1)} + \tilde{\delta}_\mu^{(b.2)} \\ a^S_{RR} &= \tilde{\delta}_\beta^{(a)} \\ a^S_{RL} = -2a^T_{RL} &= \tilde{\delta}_\beta^{(b)} \end{aligned} \tag{22}$$

where

$$\tilde{\delta}_\mu^{(a)} = \frac{\alpha M_Z^2}{\pi} \, |U_{m1}|^2 \, Z_L^{2i*} Z_L^{5i} Z_L^{1k} Z_L^{4k*} \, |N_{j1}|^2 \; \mathcal{F}_1 \left(M_{\chi^0_j}, M_{\chi^+_m}, M_{\tilde{l}_i}, M_{\tilde{l}_k} \right) \tag{23}$$

$$\tilde{\delta}_\mu^{(b.1)} = \frac{-\alpha M_Z^2}{2\pi} \, N_{j1}(N^*_{j2} - \tan\theta_W N^*_{j1}) \, N^*_{k1}(N_{k2} - \tan\theta_W N_{k1}) \times Z_\nu^{1m*} Z_\nu^{2m} Z_L^{4i*} Z_L^{5i} \; \mathcal{F}_1 \left(M_{\chi^0_j}, M_{\chi^0_k}, M_{\tilde{l}_i}, M_{\tilde{\nu}_m} \right) \tag{24}$$

$$\tilde{\delta}_\mu^{(b.2)} = \frac{-\alpha M_Z^2}{2\pi} \, N_{j1}(N_{j2} - \tan\theta_W N_{j1}) \, N^*_{k1}(N^*_{k2} - \tan\theta_W N^*_{k1}) \times Z_\nu^{1m*} Z_\nu^{2m} Z_L^{4i*} Z_L^{5i} \, M_{\chi^0_j} \, M_{\chi^0_k} \, \mathcal{F}_2 \left(M_{\chi^0_j}, M_{\chi^0_k}, M_{\tilde{l}_i}, M_{\tilde{\nu}_m} \right) \tag{25}$$

$$\tilde{\delta}_\beta^{(a)} = \frac{\alpha M_Z^2 V_{ud}}{3\pi} \, |U_{k1}|^2 \, Z_D^{1i*} Z_D^{4i} Z_L^{1m} Z_L^{4m*} \, |N_{j1}|^2 \, \mathcal{F}_1 \left(M_{\chi^0_j}, M_{\chi^+_k}, M_{\tilde{d}_i}, M_{\tilde{l}_m} \right) \tag{26}$$

$$\tilde{\delta}_\beta^{(b)} = \frac{-\alpha M_Z^2 V_{ud}}{3\pi} U_{j1} V^*_{j1} Z_U^{1i*} Z_U^{4i} Z_L^{1m} Z_L^{4m*} \, |N_{k1}|^2 \, M_{\chi^+_j} M_{\chi^0_k} \mathcal{F}_2 \left(M_{\chi^+_j}, M_{\chi^0_k}, M_{\tilde{u}_i}, M_{\tilde{l}_m} \right) \tag{27}$$

and where we have defined the loop functions

$$\mathcal{F}_n \left(m_a, m_b, m_c, m_d \right) \equiv \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \int_0^{1-x-y} \mathrm{d}z \left[x \; m_a^2 + y \; m_b^2 + z \; m_c^2 + (1-x-y-z) m_d^2 \right]^{-n} \quad . \tag{28}$$

4 Phenomenological Constraints and Implications

We now analyze the possible magnitude of the box graph contributions. At first glance, the results in Eqs. (23-27) exhibit the expected scaling with masses and couplings: $\tilde{\delta} \sim (\alpha/4\pi) \times (M_Z/\tilde{M})^2$, where $\tilde{M}$ is a generic superpartner mass. Thus, one expects these contributions to be of order 10^{-3} when $\tilde{M}$ is comparable to the electroweak scale. However, the prefactors involving products of the sfermion mixing matrices can lead to substantial departures from these expectations. (The impact of the neutralino and chargino mixing matrices N_{jk}, U_{jk} *etc.* is less pronounced). In particular, there are two general classes box graph contributions: those which depend on slepton flavor mixing ($\delta_\mu^{(b.1,2)}$) and those which depend on L-R mixing ($\delta_\mu^{(a)}$ and $\delta_\beta^{(a,b)}$). We examine each class of contributions independently by performing a numerical scan over MSSM parameter space while taking into consideration the results of direct searches for superpartners, precision electroweak data, and LFV studies. In particular, we attempt to constrain the viable parameter space – and thus the magnitude of these box graph contributions – by requiring consistency with the experimental bounds for the branching ratio for $\mu \to e\gamma$ and for $(g_\mu - 2)$.

4.1 Lepton Flavor Mixing Contributions

The contributions to g_{RR}^S from $\delta_\mu^{(b.1,2)}$ depend on the products of stermion mixing matrices $Z_\nu^{1m*} Z_\nu^{2m}$ and $Z_L^{5i} Z_L^{4i*}$, which are non-vanishing only in the presence of flavor mixing among the first two generations of sneutrinos and RH charged sleptons, respectively. The existence of such flavor mixing also gives rise to lepton flavor violating (LFV) processes such as $\mu \to e\gamma$ and $\mu \to e$ conversion and, indeed, the products $Z_\nu^{1m*} Z_\nu^{2m}$ and $Z_L^{5i} Z_L^{4i*}$ enter the rates for such processes at one-loop order. Consequently, the non-observation of LFV processes leads to stringent constraints on these products of mixing matrix elements.

To estimate the order of magnitude for these constraints, we focus on the rate for the decay $\mu \to e\gamma$, which turns out to be particularly stringent. In principle, one could also analyze the constraints implied by limits on the $\mu \to e$ conversion and $\mu \to 3e$ branching ratios. This would possibly make our conclusions on the maximal size of the flavor violating contributions to g_{RR}^S more severe, but it would not change our main conclusion: lepton flavor mixing contributions to g_{RR}^S are unobservably small.

Experimentally, the most stringent bound on the corresponding branching ratio has been obtained by the MEGA collaboration [33]:

$$Br(\mu \to e\gamma) \equiv \frac{\Gamma(\mu^+ \to e^+\gamma)}{\Gamma(\mu^+ \to e^+\nu\bar{\nu})} < 1.2 \times 10^{-11} \qquad \text{90\% C.L.} \tag{29}$$

Theoretically, a general analysis in terms of slepton and sneutrino mixing matrices has been given in Ref. [34]. Using the notation of that work, we consider those contributions to the $\mu \to e\gamma$ amplitude that contain the same combinations for LFV mixing matrices as appear in the $\tilde{\delta}_\mu^{(b)}$. For simplicity, we also set (in the present analytical estimate, but not in the following numerical computation) the chargino and neutralino mixing matrices to unity and neglect contributions that are suppressed by factors of m_μ/M_W. With these approximations, the combination $Z_\nu^{1m} Z_\nu^{2m}$ appears only in the first term of the chargino loop amplitude $A_2^{(c)R}$ according to the notation of Ref. [34]. Setting the chargino mixing to 1 (or, equivalently, considering the pure wino contribution alone), gives

$$A_2^{(c)R} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \frac{1}{m_{\tilde{\nu}_m}^2} \left(Z_\nu^{1m} Z_\nu^{2m}\right) f^{(c)}(x_m) + \cdots \ , \quad x_m = \left(\frac{m_{\tilde{\nu}_m}}{m_{\widetilde{W}}}\right) \tag{30}$$

where $f^{(c)}(x)$ is a loop function. Analogously, the combination $Z_L^{4i} Z_L^{5i}$ appears only in the first term of the neutralino loop amplitude $A_2^{(n)L}$. This time the amplitude reads (again considering only the pure *bino* loop)

$$A_2^{(n)L} \simeq \frac{\alpha}{4\pi \cos^2\theta_W} \frac{1}{m_{\tilde{L}_i}^2} \left(Z_L^{4i} Z_L^{5i}\right) f^{(n)}(x_i) + \cdots \ , \quad x_i = \left(\frac{m_{\tilde{L}_i}}{m_{\widetilde{B}}}\right) \tag{31}$$

The resulting muon decay widths respectively read

$$\Gamma^{(c)}(\mu \to e\gamma) \simeq \frac{\alpha}{4} \frac{\alpha^2}{(8\pi \sin^2\theta_W)^2} \frac{m_\mu^5}{m_{\tilde{\nu}_m}^4} \left(Z_\nu^{1m} Z_\nu^{2m}\right)^2 \left(f^{(c)}(x_m)\right)^2 \tag{32}$$

and

$$\Gamma^{(n)}(\mu \to e\gamma) \simeq \frac{\alpha}{4} \frac{\alpha^2}{(4\pi \cos^2\theta_W)^2} \frac{m_\mu^5}{m_{\tilde{L}_i}^4} \left(Z_L^{4i} Z_L^{5i}\right)^2 \left(f^{(n)}(x_i)\right)^2 \tag{33}$$

For simplicity, we consider two extremes: $m_{\tilde{\nu}_m} \approx m_{\widetilde{L}_i} \approx m_{\widetilde{W}} \approx m_{\widetilde{B}}$=100, 1000 GeV $\equiv \tilde{M}$. For either choice, we find

$$\left(f^{(c)}(1)\right)^2 \simeq \left(f^{(n)}(1)\right)^2 \simeq 0.007 \tag{34}$$

Inserting the numerical values, and requiring that $\Gamma^{(n,c)} \lesssim 10^{-30}$ GeV as required by the limit (29), we find that

$$\left(Z_L^{4i} Z_L^{5i}\right)_{\max} \approx \left(Z_\nu^{1m} Z_\nu^{2m}\right)_{\max} < \begin{cases} 10^{-3} & \text{for } \tilde{M} = 100 \text{ GeV} \\ 10^{-1} & \text{for } \tilde{M} = 1000 \text{ GeV} \end{cases} \tag{35}$$

This implies that for superpartner masses of the order of 100 GeV, the amplitudes $\tilde{\delta}_\mu^{(b.1,b.2)}$ are suppressed by a factor 10^{-6} relative to the naive expectations discussed above, while for 1000 GeV masses by a factor 10^{-2}. In this latter case, however, the loop functions in the amplitudes $\tilde{\delta}_\mu^{(b)}$ experience a further suppression factor of order 10^{-2}. Thus, the magnitude of the $\tilde{\delta}_\mu^{(b)}$ should be no larger than $\sim 10^{-7}$.

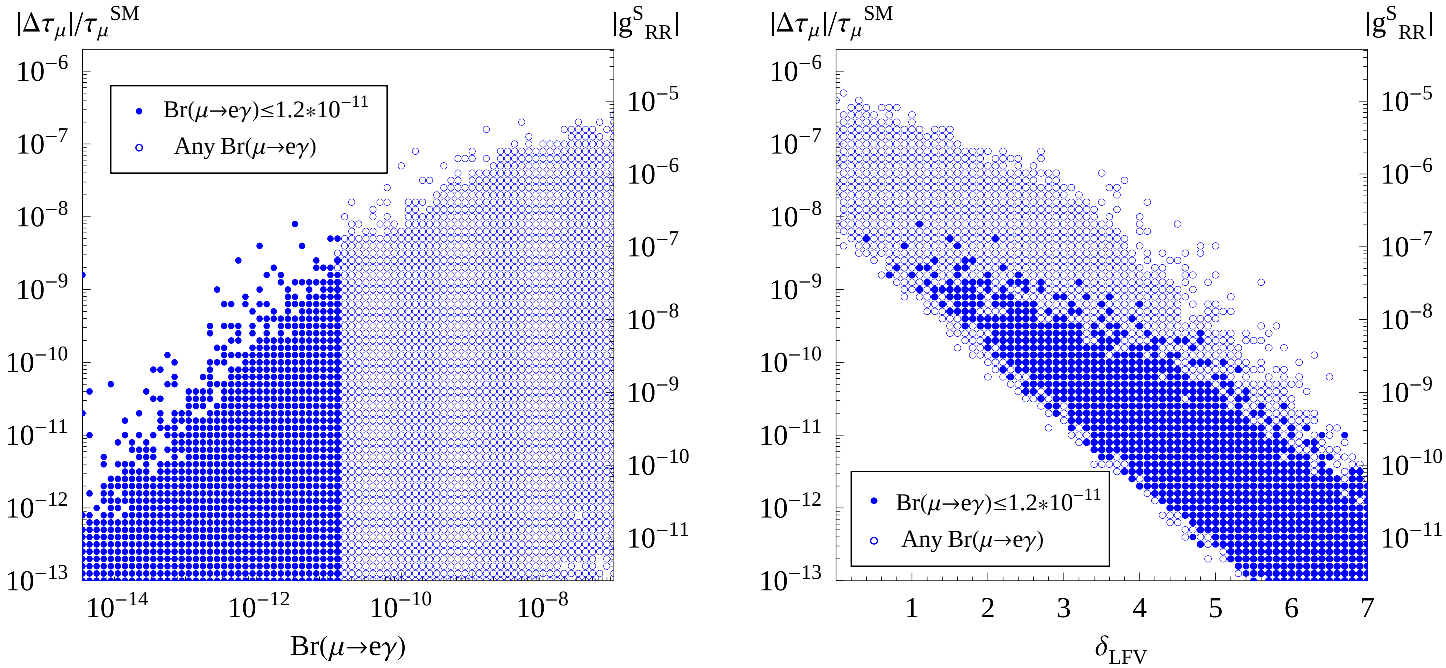
We substantiate the previous estimates by performing a numerical scan over the parameter space of the CP-conserving MSSM [35]. We do not implement any universality assumption in the slepton soft supersymmetry breaking mass sector or in the gaugino mass terms. However, in this section only, we neglect L-R mixing and consider flavor mixing between the first and second generation sleptons only. Under these assumptions, the mixing that causes non-vanishing $\tilde{\delta}_\mu^{(b.1,2)}$ stems solely from off-diagonal elements in the two 2×2 slepton mass matrices $\mathbf{M^2_{LL}}$ and $\mathbf{M^2_{RR}}$. We scan independently over all the parameters indicated in Table 1, within the specified ranges. For all models, we impose constraints from direct supersymmetric-particles searches at accelerators and require the lightest supersymmetric particle (LSP) to be the lightest neutralino (see also [36] for more details).

The result of this scan is shown in Fig. 3. Although general models can accommodate $g^S_{RR} \sim 10^{-5}$, the current constraint on $\mathrm{Br}(\mu \to e\gamma)$ severely restricts the available parameter space and reduces the allowed upper limit on g^S_{RR} by over an order of magnitude, as shown in Fig. 3 (a). It is also instructive to exhibit the sensitivity of g^S_{RR} to the degree of flavor-mixing and to show the corresponding impact of the LFV searches. To that end, we quantify the amount of lepton flavor mixing by a parameter δ_{LFV}, defined as

$$\delta_{\mathrm{LFV}} = |\delta_L| + |\delta_R| \, , \tag{36}$$

μ	m_1	m_2	$(\mathbf{M^2_{LL}})_{ij}, (\mathbf{M^2_{RR}})_{ij}$	$\tan\beta$
$30 \div 10000$	$2 \div 1000$	$50 \div 1000$	$10^2 \div 2000^2$	$1 \div 60$

Table 1: *Ranges of the MSSM parameters used to generate the models shown in Fig. 3. Here, μ is the usual higgsino mass term, while $m_{1,2}$ indicate the soft supersymmetry breaking $U(1)_Y$ and $SU(2)$ gaugino masses. The matrices $\mathbf{M^2_{LL}}$ and $\mathbf{M^2_{RR}}$ are symmetric; hence, we scanned over 6 independent masses within the specified range. All masses are in GeV.*



(a) (b)

Figure 3: *A scatter plot showing* $|\Delta\tau_\mu|/\tau_\mu^{\rm SM}$ *(left vertical axis) and* $|g^S_{\rm RR}|$ *(right vertical axis), as functions of Br*($\mu\to e\gamma$) *(a) and of* $\delta_{\rm LFV}$ *(b) (smaller* $\delta_{\rm LFV}$ *means more lepton flavor mixing; see the text for the precise definition). Filled circles represent models consistent with the current bound Br*($\mu\to e\gamma$) $\leq 1.2\times10^{-11}$*, while empty circles denote all other models.*

where

$$\delta_L = \log\left(\frac{2\,(\mathbf{M^2_{LL}})_{12}}{(\mathbf{M^2_{LL}})_{11}+(\mathbf{M^2_{LL}})_{22}}\right) \qquad\qquad \delta_R = \log\left(\frac{2\,(\mathbf{M^2_{RR}})_{12}}{(\mathbf{M^2_{RR}})_{11}+(\mathbf{M^2_{RR}})_{22}}\right) \tag{37}$$

and *e.g.*, $(\mathbf{M^2_{LL}})_{ij}$ is the (i,j)-th component of left-handed slepton mass matrix. For example, if $|\delta_L|$ is close to zero, then there is a large flavor mixing contribution from left-handed sleptons; but if $|\delta_L|$ is large, then this flavor mixing is suppressed. Since the amplitudes $\tilde{\delta}_\mu^{(b.1,2)}$ depend on flavor mixing among *both* LH and RH sleptons, they contribute only if both δ_L and δ_R are small. (In contrast, Br($\mu\to e\gamma$) survives in the presence of flavor mixing among *either* LH or RH sleptons.) Naturally, the flavor mixing contribution to g^S_{RR} is largest when δ_{LFV} is smallest, as shown in Fig. 3(b). We note that to obtain a large flavor mixing contribution to g^S_{RR}, it is not sufficient simply to have maximal mixing (i.e. $|Z_L^{ij}| = 1/\sqrt{2}$); in addition, one needs the absence of a degeneracy among the slepton mass eigenstates, or else the sum over mass eigenstates [*e.g.*, sum over i and i' in Eqn (24)] will cancel. In any case, we conclude that the flavor mixing box graph contributions are too small to be important for the interpretation of the next generation muon decay experiments, where the precision in τ_μ is expected to be of the order of one ppm.

4.2 Left-Right Mixing Contributions

Significantly larger contributions to g^S_{RR} can arise from $\tilde{\delta}^{(a)}_\mu$, which requires only L-R mixing among same generation sleptons. In the case of smuon L-R mixing, some considerations follow from the present value for the muon anomalous magnetic moment, or $(g_\mu - 2)$, which is a chirality odd operator and which can arise from L-R mixing in one-loop graphs [37]. The only supersymmetric contribution δa_μ to $a_\mu \equiv (g_\mu - 2)/2$ proportional to one single power of the ratio of the muon mass and of supersymmetric particles is in fact proportional to the smuon L-R mixing, and reads

$$\delta a^{\rm LR-mix}_\mu = \frac{m_\mu}{16\pi^2}\sum_{i,m}\frac{m_{\chi^0_i}}{3m^2_{\tilde{\mu}_m}}{\rm Re}[g_1 N_{i1}(g_2 N_{i2} + g_1 N_{i1}) Z_L^{2m*} Z_L^{5m}] F_2^N(m^2_{\chi^0_i}/m^2_{\tilde{\mu}_m}), \tag{38}$$

where F_2^N is the appropriate loop function specified in [37]. The expression features the same dependence upon the smuon mixing matrix as does Eq. (23). Under the widely considered alignment assumption that $\mathbf{a_f} \propto \mathbf{Y_f}$ this term is usually suppressed, as the smuon L-R mixing is also suppressed. Here, however, we drop that hypothesis, and allow for large L-R mixing: this, in general, enhances the aforementioned contribution, and we therefore expect that the experimental constraints on $(g_\mu - 2)$ will set limits on g^S_{RR}. In particular, assuming a common mass $\tilde{M}$ for all the supersymmetric particles, and maximal L-R mixing, Eq. (38) approximately reduces to

$$\delta a^{\rm LR-mix}_\mu \approx \frac{g_1^2}{4\pi}\frac{1}{12\pi}\left(\frac{m_\mu}{\tilde{M}}\right) \approx 10^{-7} \quad \text{for } \tilde{M} \sim 100 \text{ GeV}, \tag{39}$$

where g_1 indicates the $U(1)_Y$ gauge coupling. We consider here the 95% C.L. limit on beyond-the-SM contributions to $(g_\mu - 2)$ as quoted in Ref. [38],

$$a^{\rm exp}_\mu - a^{\rm th-SM}_\mu = (25.2 \pm 9.2) \times 10^{-10}, \tag{40}$$

bearing in mind that a more conservative approach to the evaluation of the sources of theoretical uncertainty in the SM contribution could inflate the error associated with $a^{\rm th-SM}_\mu$ (see *e.g.* Ref. [39] for the hadronic light-by-light contribution).

A large value for δa_μ, however, does not imply automatically a large g^S_{RR}, since the latter also depends upon the mixing in the selectron sector, to which δa_μ is blind. On the other hand, large values of g^S_{RR} in general should produce a sizable δa_μ, although the possibility of cancellations with other terms, and the different loop function structures can lead, in principle, to a suppression of δa_μ even for large g^S_{RR}.

We illustrate how g^S_{RR} depends upon the degree of L-R mixing, and how it correlates with δa_μ, in Figure 4. For this figure we set $\tan\beta = 5$, $M^2_{LL}(\mu) = (100 \text{ GeV})^2$, $M^2_{RR}(\mu) = (300 \text{ GeV})^2$, $m_1 = m_2/2 = 100$ GeV and $\mu = 200$ GeV, and use as the independent variable the quantity $\sqrt{M^2_{LR}(\mu)}/\text{GeV}$. We show with a red dashed line the loop function factor $M_Z^2\mathcal{F}_1$ in Eq. (28), with

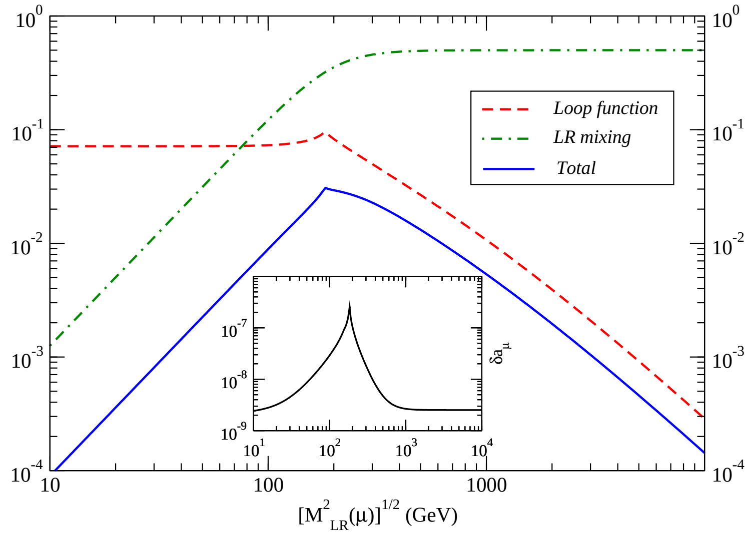


Figure 4: *A sketch of the behavior of* $g^S_{\rm RR}$ *as a function of* $\sqrt{M^2_{LR}(\mu)}/GeV$*. The red dashed line indicates the loop function factor:* $M^2_Z\mathcal{F}_1$ *in Eq. (28); the green dashed-dotted line the L-R mixing factor:* $|(Z^{22}_L)^* Z^{52}_L|$*), while the solid blue line is the product* $M^2_Z\mathcal{F}_1\times|(Z^{22}_L)^* Z^{52}_L|$*. The left-hand axis scale refers to the red dashed and blue lines, while the right-hand axis scale to the L-R mixing. In the inset we show, again as a function of*$\sqrt{M^2_{LR}(\mu)}/GeV$*, the supersymmetric contribution to* $(g_\mu-2)$*. In the figure, we employed* $\tan\beta=5$*,* $M^2_{LL}(\mu)=(100\ \mathrm{GeV})^2$*,* $M^2_{RR}(\mu)=(300\ \mathrm{GeV})^2$*,* $m_1=m_2/2=100\ GeV$ *and* $\mu=200\ GeV$*.*

a green dashed-dotted line the L-R mixing factor $|(Z^{22}_L)^* Z^{52}_L|$, and, finally, with a solid blue line the product$M^2_Z\mathcal{F}_1\times|(Z^{22}_L)^* Z^{52}_L|$. In the inset we show the corresponding supersymmetric contribution to the muon anomalous magnetic moment, again as a function of $\sqrt{M^2_{LR}(\mu)}$/GeV.

The quantity g^S_{RR} is proportional to the quantity shown with the blue line, and features a maximum when one of the two eigenvalues of the smuon mass matrix is driven to very small values, corresponding to $M^2_{LR}(\mu)\approx\sqrt{M^2_{LL}(\mu)\times M_{RR}(\mu)^2}$. A very light smuon enhances both the loop function and δa_μ; a further enhancement to δa_μ comes however also from L-R mixing, which is maximal when $M^2_{LR}(\mu)\gg\sqrt{M^2_{LL}(\mu)\times M_{RR}(\mu)^2}$. Increasing $M^2_{LR}(\mu)$ eventually saturates the relation $m_{\tilde\mu_{1,2}}\simeq\sqrt{M^2_{LR}(\mu)}$, and leads to a suppression of both g^S_{RR} (through the loop function) and of δa_μ (which asymptotically reaches the value of the chargino-sneutrino contribution alone, independent of the L-R mixing in the smuon sector). Note that for this choice of parameters, most of the peak region in the inset is excluded by the $(g_\mu-2)$ results. Thus, one would expect $|g^S_{RR}|$ to be no larger than $\sim 10^{-5}$ for values of $\sqrt{M^2_{LR}(\mu)}$ lying outside this peak region (recall that the magnitude of g^S_{RR} is roughly the value given by the solid blue curve in Fig. 4 times α/π). However, larger values can be obtained consistent with the $(g_\mu-2)$ results for other parameter choices (see Fig. 5 below).

We also observe that for a given supersymmetric parameter space setup, there exists a special value of the trilinear scalar coupling entering $M^2_{LR}(\mu)$ that maximizes g^S_{RR}. This will correspond to the trade off between the product of the red and the green lines in Fig. 4, *i.e.* between the loop function effect and the size of the L-R mixing. The same is true for the case of supersymmetric contributions to β decay, where again one can find the values of the relevant trilinear scalar couplings such that $\tilde{\delta}_\beta$ is maximal.

The possible existence of nearly flat directions in the MSSM potential leads to constraints on the size of the scalar trilinear couplings from the condition of avoiding charge and color breaking minima. Quantitatively, one can express those constraints in the form [40]

$$\begin{aligned} \mathbf{a_u^2} &\lesssim 3 \times \left(\mu_u^2 + \mathbf{m^2_{\tilde{Q}}} + \mathbf{m^2_{\tilde{u}}}\right) \\ \mathbf{a_d^2} &\lesssim 3 \times \left(\mu_d^2 + \mathbf{m^2_{\tilde{Q}}} + \mathbf{m^2_{\tilde{d}}}\right) \\ \mathbf{a_e^2} &\lesssim 3 \times \left(\mu_d^2 + \mathbf{m^2_{\tilde{L}}} + \mathbf{m^2_{\tilde{e}}}\right) \end{aligned} \qquad (41)$$

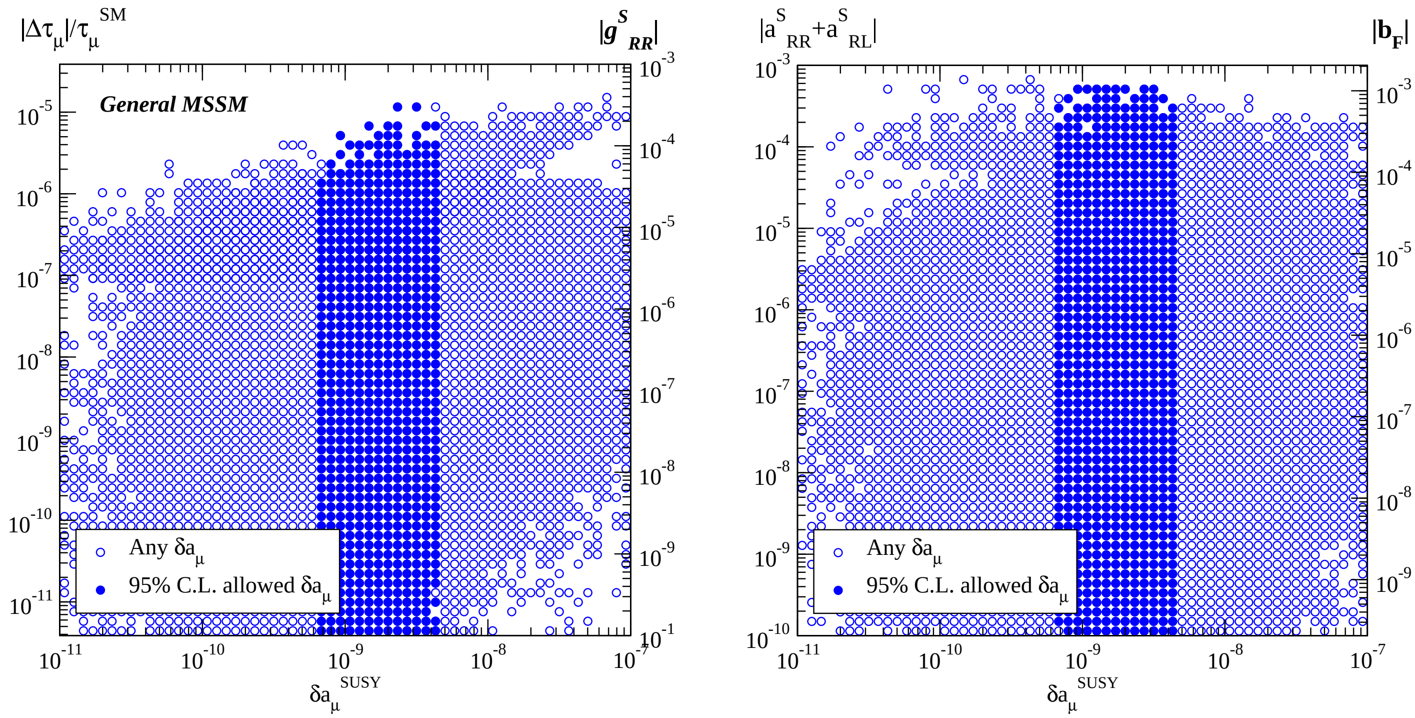
where $\mu^2_{u,d} \equiv m^2_{h_{u,d}} + |\mu|^2$. Making use of the electroweak symmetry breaking (EWSB) conditions, one can express $\mu^2_{u,d}$ as functions of M_Z and of the CP-odd Higgs mass, m_A, and – at the expense of introducing some fine-tuning – achieve arbitrarily large values for the size of the trilinear scalar couplings, as long as m_A is sufficiently large, independently of the other soft supersymmetry breaking mass terms. While this argument cannot be applied to models such as minimal supergravity, where the sfermion and the heavy Higgs sector are connected at the GUT scale, in extended models (such as, for instance, the non-universal Higgs mass extension of mSUGRA, [41]) the size of the trilinear scalar couplings can be taken to be much larger than the size of the soft breaking sfermion mass terms. In turn, this implies the possibility of having a sizable L-R mixing not only in the third generation sfermions but, in principle, in the first two generations as well.

The box graph contributions to a^S_{RR} and $a^{S,T}_{RL}$ live entirely on L-R mixing among first generation sleptons and squarks. To our knowledge, there exist no strong bounds on such mixing from precision electroweak measurements or searches for rare or SM-forbidden processes. Consequently, we will consider the possibility of maximal L-R mixing which simply requires that $|M^2_{LR}| \sim |M^2_{LL} - M^2_{RR}|$. From Eqs. (14-18), this situation amounts to having a_f of order the electroweak scale and m^2_F not too different from $m^2_{\tilde{f}}$.

Taking into account the foregoing considerations, we carry out a numerical analysis of the mag-

μ	m_1	m_2	m_3	m_A	$m_{\widetilde{F}}$	A_F	$\tan\beta$
$30 \div 10000$	$2 \div 1000$	$50 \div 1000$	$m_{\text{LSP}} \div 10000$	$100 \div 10000$	$(1 \div 10) m_{\text{LSP}}$	$\pm(m^2_{\widetilde{F}}/m_F)$	$1 \div 60$

Table 2: *Ranges of the MSSM parameters used to generate the models shown in Figs. 5 and 6. All masses are in GeV, and* $m_{\text{LSP}} \equiv \min(|\mu|, |m_1|, |m_2|)$. m_3 *and* m_A *indicate the gluino and the CP odd heavy Higgs boson masses, respectively. The quantity* $m_{\widetilde{F}}$ *indicates the various soft supersymmetry breaking masses, which we independently sampled; we dub the mass of the corresponding SM fermion* F *as* m_F.

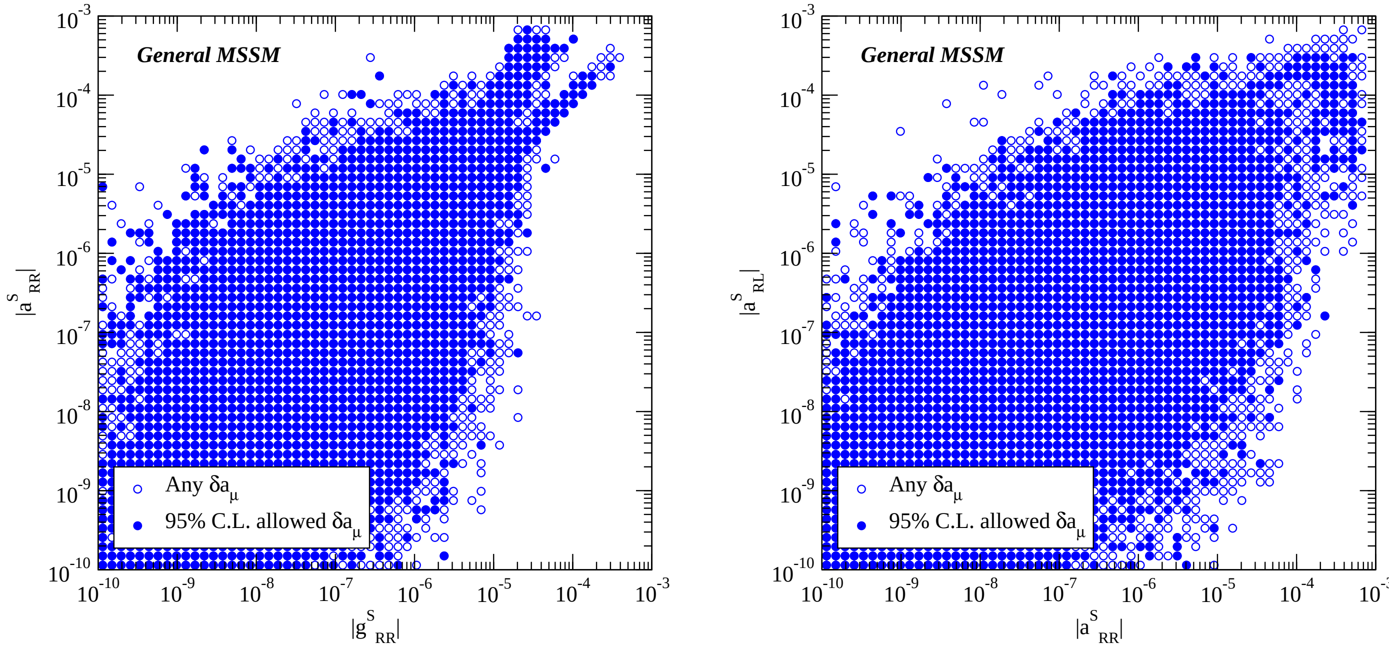


(a) (b)

Figure 5: *A scatter plot showing* $|\Delta\tau_\mu|/\tau_\mu^{\rm SM}$ *(a) and* $|g^S_{\rm RR}|$ *(b), relative to muon decay, left, and* $a^S_{\rm RR}+a^S_{\rm RL}$*, relative to* β *decay, right, as a function of the supersymmetric contribution to the muon anomalous magnetic moment* δa_μ*. Filled circles represent models consistent with the current 95% C.L. range for beyond the standard model contributions to* $(g_\mu-2)$*, while empty circles denote all other models.*

nitude of the SUSY contributions to g^S_{RR}, a^S_{RR}, and $a^{S,T}_{RL}$. As before, we consider the CP-conserving MSSM [35], and proceed to a random scan over its parameter space. We do not resort to any universality assumption, neither in the scalar soft supersymmetry breaking mass sector, nor in the gaugino mass terms nor in the soft-breaking trilinear scalar coupling sector, and scan independently over all the parameters indicated in Table 2, within the specified ranges. We indicate with $m_{\widetilde F}$ a generic scalar fermion soft mass (corresponding to a standard model fermion whose mass is m_F), and with $m_{\rm LSP}$ the smallest mass parameter entering the neutralino mass matrix (namely, m_1, m_2 and μ), in absolute value. For all models, we impose constraints from direct supersymmetric-particles searches at accelerators, rare processes with a sizable potential supersymmetric contribution, the lower bound on the mass of the lightest CP-even Higgs boson, and precision electroweak tests. We also require the lightest supersymmetric particle (LSP) to be the lightest neutralino (see also [36] for more details).

We show the results of our scan in Fig. 5-7. In particular, we indicate in Fig. 5, (a), the values of $|\Delta\tau_\mu|/\tau_\mu^{\rm SM}$ (left axis) and $|g^S_{\rm RR}|$ (right axis) we obtained in our scan, as a function of the supersymmetric contribution to the muon anomalous magnetic moment, δa_μ. Filled circles represent models consistent



(a) (b)

Figure 6: *The correlation between* $a^S_{\rm RR}$ *and* $g^S_{\rm RR}$ *(a) and between* $a^S_{\rm RR}$ *and* $a^S_{\rm RL}$ *(b). Filled circles represent models consistent with the current 95% C.L. range for beyond the standard model contributions to* $(g_\mu - 2)$*, while empty circles denote all other models.*

with the current 95% C.L. range for beyond the standard model contributions to $(g_\mu - 2)$, while empty circles denote all other models. As we anticipated, large values of δa_μ do not always imply large $|g^S_{\rm RR}|$, and, vice-versa. The values of $|g^S_{\rm RR}|$ compatible with the limits on $(g_\mu - 2)$ and with all constraints on the supersymmetric setup can be as large as a few times 10^{-4}, though the size of the effect could also be many orders of magnitude smaller. The models giving the largest effects tend to have large L-R mixing (and hence large trilinear scalar couplings) both in the smuon and in the selectron spectrum, and, naturally, a light supersymmetric particle spectrum. In contrast, assuming alignment between the triscalar and Yukawa matrices leads to unobservably small effects in μ-decay.

Current limits on the parameter g^S_{RR} obtained from direct studies of μ-decay observables lead to an upper bound of 0.067 according to the recent global analysis of Ref. [42]. Thus, improvements in precision by more than two orders of magnitude would be required to probe these non-$(V-A)\otimes(V-A)$ contributions in the large L-R mixing regime. On the other hand, the impact of g^S_{RR} on the extraction of G_μ from the muon lifetime could become discernible at the level of precision of the muon lifetime measurements underway at PSI [10, 11]. These experiments expect to improve the precision on τ_μ such that the experimental error in G_μ is 10^{-6}. At such a level, a contribution to η from g^S_{RR} of order 10^{-4} would begin to be of interest, as per Eq. (4). In particular, we note that there exist regions of the MSSM parameter space that generate contributions to $\Delta\tau_\mu/\tau_\mu$ as large as a few $\times 10^{-6}$ via the η parameter in Eq. (4), corresponding to $\sim$ ppm corrections to G_μ.

Consideration of this correction could be particularly interesting if future measurements at a facility such as GigaZ lead to comparable improvements in other electroweak parameters, such as M_Z and $\sin^2\hat\theta_W(M_Z)$. A comparison of these quantities can provide a test of the SM (or MSSM) at the level of electroweak radiative corrections via the relation [43]

$$\sin^2\hat\theta_W(M_Z)\cos^2\hat\theta_W(M_Z) = \frac{\pi\alpha}{\sqrt{2}M_Z^2 G_\mu\left[1-\Delta\hat r(M_Z)\right]} \tag{42}$$

where $\Delta\hat r(M_Z)$ contains electroweak radiative corrections to the $(V-A)\otimes(V-A)$ μ-decay amplitude, the Z-boson self energy, and the running of $\hat\alpha$. Any discrepancy in this relation could signal the presence of new physics contributions to $\Delta\hat r(M_Z)$ beyond those obtained in the SM (or MSSM). Inclusion of ppm corrections to G_μ arising from the presence of a non-zero η in Eq. (4) would be important in using Eq. (42) to carry out a ppm self-consistency test. Resolution of other theoretical issues in the computation of $\Delta\hat r(M_Z)$ – such as hadronic contributions to the running of $\hat\alpha$ – would also be essential in performing such a test.

In the case of β-decay, we show the analogue of the figure described above for the muon decay, in Fig. 5, (b). We show, as a function of δa_μ, the value of $a_{RR}^S + a_{RL}^S$. We find that values of $a_{RR}^S + a_{RL}^S$ as large as 10^{-3} are consistent with all phenomenological constraints. Since the amplitudes $\tilde\delta_\beta^{(a,b)}$ depend on L-R mixing among first, rather than second, generation sleptons and squarks (the factors $Z_L^{1i'} Z_L^{4i'\,*}$ and $Z_Q^{1i\,*} Z_L^{4i}$, $Q = U$ or D, respectively) the parameters a_{RR}^S, a_{RL}^S, and a_{RL}^T are not as constrained by precision measurements as is g_{RR}^S. Thus, it is possible for the β-decay parameters to reach their naive, maximal scale $\alpha/4\pi$ in the limit of maximal L-R mixing.

The correlations between g_{RR}^S and a_{RR}^S, and between a_{RR}^S and a_{RL}^S, are shown in the panels (a) and (b), respectively, of Fig. 6. In the figure, again, filled circles represent models consistent with the current 95% C.L. range for beyond the standard model contributions to $(g_\mu - 2)$, while empty circles denote all other models. We notice that in general there exists no strong correlation among the various quantities, ane we find some correlation only for very large values of the quantities under investigation, in the upper right portions of the plots. Also, no hierarchy between a_{RR}^S and a_{RL}^S exists.

A correlation between the various quantities of interest does arise, however, when some priors are in place on the supersymmetric particle spectrum. In particular, we show the results of a scan of minimal supergravity models (where scalar soft breaking mass universality is imposed at the GUT scale) in Fig. 7 (a). In particular, we display on the vertical axis the values of $a_{RR}^S + a_{RL}^S$, and on the horizontal axis g_{RR}^S, for models where we imposed a maximal L-R mixing (see Fig. 4). In this case, we obtain that $0.7 \lesssim (a_{\rm RR}^S + a_{\rm RL}^S)/g_{\rm RR}^S \lesssim 1.5$. In contrast to the model-independent parameter space scans, a nearly linear correlation between these parameters arises due to the mSUGRA-dependent relations between sfermion masses and requirement of maximal L-R mixing. Moreover, the magnitude

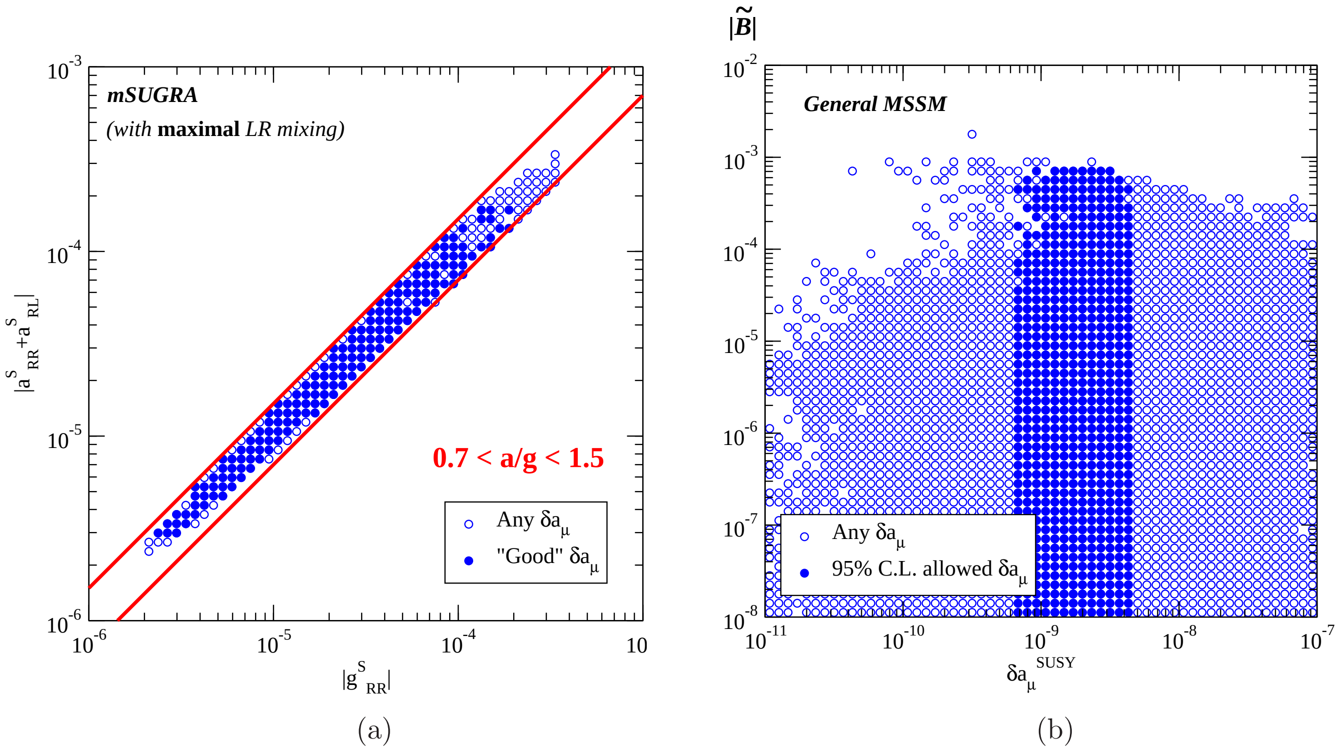


Figure 7: (a): *The correlation between* $a^S_{\rm RR}+a^S_{\rm RL}$ *and* $g^S_{\rm RR}$ *in "*minimal supergravity*" models with maximal left-right mixing (i.e., where the trilinear scalar couplings have been set to the values corresponding to a maximal contribution to the quantities of interest, see Fig. 4). The scalar mass universality condition at the GUT scale dictates the strong correlation between the two quantities* $0.7 \lesssim (a^S_{\rm RR}+a^S_{\rm RL})/g^S_{\rm RR} \lesssim 1.5$. *Again, filled circles represent models consistent with the current 95% C.L. range for beyond the standard model contributions to* $(g_\mu-2)$*, while empty circles denote all other models.* (b): *MSSM-induced non-*$(V-A)\otimes(V-A)$ *contributions to the energy-dependence of the* β*-decay neutrino asymmetry parameter,* B*. Here, we have scaled out the energy-dependence and have plotted* $\tilde{B}=B/(\Gamma m/E)$ *for various randomly generated MSSM parameters. As before, dark circles indicate models consistent with* $(g_\mu-2)$ *We have also assumed* $g_S/g_V=1=g_T/g_A$

of the β-decay couplings is generally less than $\lesssim 10^{-4}$ due to the $(g_\mu-2)$ constraints on smuon masses and the mSUGRA sfermion mass relations. It is interesting to note that within this model scenario, the observation of a non-zero β-decay correlation at the $\sim 10^{-4}$ level would imply a non-zero g^S_{RR} of similar magnitude, along with the corresponding correction to the theoretical μ-decay rate.

In the more general, model-independent situation, it is important to emphasize that large L-R mixing in the first generation slepton and squark sectors can lead to a^S_{RR}, a^S_{RL}, and a^T_{RL} as large as $\mathcal{O}(10^{-3})$. Coefficients of this magnitude could, in principle, be probed with a new generation of precision β-decay correlation studies. At present, the most precise tests of these quantities arises from superallowed Fermi nuclear β-decay, from which one obtains constraints on the Fierz interference coefficient $b_F = 0.0026(26)$ [44, 45]. For this transition one has

$$b_F = \pm \frac{2\, g_S}{g_V} \frac{a^S_{RL} + a^S_{RR}}{a^V_{LL}} \tag{43}$$

independent of the details of the nuclear matrix elements[4]. In Fig. 5, (b), we also show the quantity b_F assuming $g_S/g_V = 1$ in the right-hand vertical axis. The present experimental sensitivity lies just above the upper end of the range of possible values of b_F.

It is also interesting to consider the recent global analysis of Ref. [24], where several different fits to β-decay data were performed. The fit most relevant to the presence analysis corresponds to "case 2" in that work, leading to bounds on the following quantities:

$$\begin{aligned} R_S &\equiv \frac{g_S}{g_V} \frac{a^S_{RL} + a^S_{RR}}{a^V_{LL}} \\ R_T &\equiv \frac{2\, g_T}{g_V} \frac{a^T_{RL}}{a^V_{LL}} \end{aligned} \tag{44}$$

In particular, including latest results for the neutron lifetime [46] that differs from the previous world average by six standard deviations leads to a non-zero R_T: $R_T = 0.0086(31)$ and $R_S = 0.00045(127)$ with $\chi^2/\text{d.o.f.} = 1.75$. In contrast, excluding the new τ_n result implies both tensor and scalar couplings consistent with zero. We note that SUSY box graphs could not account for tensor couplings of order one percent since the natural scale of the relevant correction – $\tilde{\delta}^{(b)}_\beta$ – is $\alpha/2\pi \sim 0.1\%$ in the case of maximal L-R mixing and SUSY masses of order M_Z [see Eq. (27)]. Moreover, there exist no logarithmic or large $\tan\beta$ enhancements that could increase the magnitude of this amplitude over this scale.

Future improvements in experimental sensitivity by up to an order of magnitude could allow one to probe the MSSM-induced non-$(V-A)\otimes(V-A)$ contributions to the β-decay correlation coefficients in the regime of large L-R mixing. For example, future experiments using cold and ultracold neutrons could allow a determination of the energy-dependent component of the neutrino asymmetry parameter B in polarized neutron decay at the level of a few $\times 10^{-4}$. As indicated by the scatter plot in Fig. 7 (b) – where we show the range of values for the energy-dependent part of the neutrino asymmetry – experiments with this level of sensitivity could probe well into the region of parameter space associated with large L-R mixing [47]. Similarly, prospects for significant improvements in the sensitivity to the Fierz interference term using nuclear decays at a new radioactive ion beam facility are under active consideration [48].

As with other low-energy, semileptonic observables, the theoretical interpretation of the β-decay correlation coefficients requires input from hadron structure theory. For example, the form factors that multiply the scalar and tensor couplings have not been determined experimentally, and there exists some latitude in theoretical expectations for these quantities. The current estimates are [22]

$$0.25 \lesssim g_S \lesssim 1 \qquad\qquad 0.6 \lesssim g_T \lesssim 2.3;. \tag{45}$$

[4]Here, we have assumed all quantities are relatively real.

These ranges derive from estimates for neutral current form factors assuming the quark model and spherically-symmetric wavefunctions [49]. In obtaining the dependence of b_F and B on MSSM parameters as in Figs. 5,7, we have assumed $g_S/g_V = 1 = g_T/g_A$, so the final sensitivities of correlation studies to MSSM-induced non-$(V-A)\otimes(V-A)$ interactions will depend on firm predictions for these ratios. Similarly, the effects of second class currents generated by the small violation of strong isospin symmetry in the SM may generate β energy-dependent contributions to the correlation coefficients that mimic the effects of the MSSM-induced scalar and tensor interactions discussed here. An analysis of these effects on the correlation coefficients a and A has been recently performed in Ref. [50]. To our knowledge, no such study has been carried out for the correlation coefficients of interest here. Carrying out such an analysis, as well as sharpening the theoretical expectations of Eq. (45), would clearly be important for the theoretical interpretation of future correlation studies.

5 Discussion and Conclusions

If supersymmetric particles are discovered at the LHC, it will be then important to draw predictions on a wide array of observables in order to determine the parameters that describe the superpartners interactions. As we have discussed above, precision studies of weak decay correlations may provide one avenue for doing so. In particular, such studies could probe a unique feature of SUSY not easily accessed elsewhere, namely, triscalar interactions involving first and second generation scalar fermions. The presence of triscalar interactions is implied by both purely supersymmetric Yukawa and bilinear components of the superpotential and by soft, SUSY-breaking triscalar interactions in the Lagrangian. The flavor and chiral structure of the latter are particularly vexing, since – in the MSSM – one has both a large number of *a priori* unknown parameters and – experimentally – a limited number of handles with which to probe them.

In light of this situation, it has been the common practice to rely on models that relate various parameters and reduce the number of inputs that must be determined from data. Conventionally, one makes the "alignment" assumption, wherein the soft-triscalar couplings for a given species of fermion are proportional to the corresponding Yukawa matrices. Under this assumption, one would expect the effects of soft triscalar interactions to be suppressed for the first and second generations. While the supersymmetric triscalar interactions are, indeed, proportional to the Yukawa couplings, the soft triscalar interactions need not be. As we have argued above, the study of weak decay correlations offer a means for testing this possibility experimentally.

The effects of triscalar couplings in weak decay correlations arise from one-loop graphs that generate scalar and tensor interactions. These interactions are forbidden in the SM CC interaction in the limit of massless fermions since it involves only LH fermions and since the scalar and tensor operators couple fields of opposite chirality. In the MSSM, such terms can arise via L-R mixing of virtual scalar fermions in one-loop box graphs, and this L-R mixing can be significant when the corresponding soft, triscalar couplings are unsuppressed. In the case of μ-decay, additional contributions to scalar and tensor four-fermion operators can also be generated by flavor-mixing among same-chirality scalar leptons, but this flavor-mixing is highly constrained by LFV studies such as $\mu \to e\gamma$. Thus, for both μ- and β-decay, observable, SUSY-induced scalar and tensor couplings can only be generated by flavor diagonal L-R mixing.

Probing these interactions would require improvements in precision of one- and two-orders of magnitude, respectively, for β-decay and μ-decay correlation coefficients. Order of magnitude improvements for β-decay appear realistic, while the necessary advances for μ-decay appear to be more daunting. On the other hand, if SUSY is discovered at the LHC, then considerations of SUSY-induced, four-fermion scalar interactions involving RH charged leptons may become necessary when extracting the Fermi constant from the muon lifetime. Doing so could become particularly important when ppm tests of electroweak symmetry become feasible.

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